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ST. SAINT'S COLLEGE LIBRARY TRIVANDRUM

N – 7745

Reg. No. : .....

Name : .....

Fourth Semester B.Sc. Degree Examination, August 2022

First Degree Programme under CBCSS

Mathematics

Core Course III

MM 1441 : ALGEBRA AND CALCULUS II

(2014 – 2017 Admission)

Max. Marks : 80

Time : 3 Hours

SECTION – I

All the first **ten** questions are compulsory. They carry **1** mark each.

1. Find $p([2])$ in $Z/6Z$, if $p(x) = [3]x + [4]x^3$.
2. Give two different polynomials with coefficients in $Z/2Z$ which agree as functions.
3. State Remainder theorem.
4. Define irreducible polynomial with an example.
5. Find the remainder in $\mathbb{Q}[x]$ when $x^3 - 7x - 1$ is divisible by $x - 2$.
6. Find $f(e, 0)$ and the natural domain of $f(x,y) = \sqrt{(y+1)} + \ln(x^2 - y^2)$.
7. Evaluate $\lim_{(x, y)\to(0, 0)} \ln(1 + x^2y^3)$.

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8. If $f(x, y) = x \sin xy$, find $f_x(x, y)$.

9. Evaluate $\int_0^1 \int_{-3}^{2} y^2 x \, dx \, dy$.

10. Define critical point of a function $f(x, y)$

(10 × 1 = 10 Marks)

SECTION – II

Answer any **eight** questions. They carry **2** marks each.

11. Detach the coefficients and multiply $(x^3 + 1)(2x^2 + 2)$ in $\mathcal{F}_3[x]$.

12. For which values of k in $\mathcal{Q}$ does $x - k$ divide $x^3 - kx^2 - 2x - k + 3$?

13. Find a greatest common divisor of $x^2 - 1$ and $5x^2 + 10x + 5$ in $\mathcal{Q}[x]$.

14. Write x^4 in base $x + 2$.

15. Find the quotient and remainder when $x^4 - 2x^2 - 1$ is dividend by $x^2 + 3x - 1$ in $\mathcal{Q}[x]$.

16. Show that $1 + 2x$ is a unit of $Z/4Z[x]$.

17. Evaluate $\lim_{(x, y)\to(0, 0)} (x^2 + y^2) \sin\left(\frac{1}{x^2 + y^2}\right)$.

18. Describe the level surface $f(x, y, z) = (x - 2)^2 + y^2 + z^2$ in words.

19. If $(x^2 + y^2 + z^2)^{\frac{3}{2}} = 1$, calculate $\frac{\partial z}{\partial x}$ and $\frac{\partial z}{\partial y}$ using implicit differentiation.

20. Evaluate $\int_1^{\frac{3}{2}} \int_y^{3-y} y \, dx \, dy$.

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21. Use a polar double integral to find the area enclosed by the three-petaled rose $r = \sin 3\theta$.

22. Find parametric equations for the portion of the cylinder $x^2 + y^2 = 5$ that extends between the planes $z = 0$ and $z = 1$.

(8 × 2 = 16 Marks)

SECTION – III

Answer any **six** questions. They carry **4** marks each.

23. Prove that a nonzero polynomial $f(x)$ of degree n in $\mathcal{F}[x]$, where $\mathcal{F}$ is a field, has at most n distinct roots in $\mathcal{F}$.

24. Prove that for any n, $\sum_{\frac{d}{n}} \phi(d) = n$.

25. Show that if p is irreducible, and f is any polynomial which is not divisible by p, then the greatest common divisor of p and f is 1.

26. Find the gcd of $x^6 - 1$ and $x^4 - 1$ in $\mathbb{Q}[x]$ and express it as in Bezout's identity.

27. Find all solutions of $x^3 = 7x + 6$ using Cardano's method.

28. If $f(x,y) = 4x^2 - 2y + 7x^4y^5$, find (a) f_{xx} (b) f_{yy} (c) f_{xy} (d) f_{yx}.

29. Determine whether the limit $\lim_{(x,\,y)\to(0,\,0)} \frac{xy}{3x^2 + 2y^2}$ exist. Justify your answer.

30. Given that $z = e^{xy}$, $x = 2u + v$, $y = \frac{u}{v}$, find $\frac{\partial z}{\partial u}$ and $\frac{\partial z}{\partial v}$ using the chain rule.

31. Evaluate $\int_0^3 \int_0^{\sqrt{9-z^2}} \int_0^x xy \, dy \, dx \, dz$.

(6 × 4 = 24 Marks)

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SECTION – IV

Answer any **two** questions. They carry **15** marks each.

32. (a) Show that if $f(x)$ in $\mathcal{R}[x]$ has a multiple factor, then its derivative $f'(x)$ is not relatively prime to $f(x)$.

(b) Using trial division in $Z/2Z[x]$, factor $x^6 + x^4 + x$ into a product of irreducible polynomials.

33. State and prove Division Theorem.

34. (a) Find the absolute extrema of the function $f(x, y) = xy - x - 3y$ on the triangular region with vertices (0, 0), (0, 4) and (5, 0).

(b) Use Lagrange multipliers to find the maximum and minimum values of $f(x, y) = 4x^3 + y^2$ subject to the constraint $2x^2 + y^2 = 1$. Also, find the points at which these exteme values occur.

35. (a) Find the surface area of that portion of the surface $z = \sqrt{4 - x^2}$ that lies above the rectangle R in the xy–plane whose coordinates satisfy $0 \le x \le 1$ and $0 \le y \le 4$.

(b) Find the volume of the solid enclosed between the paraboloids $z = 5x^2 + 5y^2$ and $z = 6 - 7x^2 - y^2$.

(2 × 15 = 30 Marks)

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